SEASONAL ADJUSTMENT

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1. Introduction

Socio-economic time series data are used for studying and following the developments of trends and for detecting the occurrence of turning points and changes in the direction of the socio-economic activity. This detection task is made difficult when the original data contain not only the fundamental trend-cycle behavior of interest, but also the movements attributable to seasonal, trading day, movable holiday and irregular influences. In order to estimate the trend in time series, seasonal, movable holiday and trading day variations must first be removed and then irregular influences dampened.

This publication describes the latest methods used for seasonal adjustment at the Central Bureau of Statistics (CBS). We also provide here comprehensive information regarding more than 400 time series that are regularly published by the Bureau. The publication is based on the research on time series analysis carried out by the Statistical Analysis Sector. In this research, methods developed in other leading statistical agencies, academic institutions and central banks around the world are analyzed, new methods are developed, and those appropriate to Israeli time series are applied. In particular, the prior adjustment factors are calculated using the special method developed by the CBS for the estimation of the moving Jewish festival dates and trading day effects in Israel. The seasonal factors, and the seasonally adjusted series, are calculated using the X-12-ARIMA (U.S. Census Bureau, 2001) seasonal adjustment method. The estimation of the trend is carried out according to an improved method based on symmetric Henderson moving averages and an additional process that is described in Section 4.3.1.

The first part of the publication focuses on a detailed presentation of our methodology. In Section 2 we define a time series and its components. In Section 3 we describe the X-11-ARIMA and X-12-ARIMA seasonal adjustment methods. X-12-ARIMA is the latest in the family of the seasonal adjustment methods developed over several decades by the U.S. Census Bureau and Statistics Canada. In 2002, we started the process of implementing this method as the standard program. Since the beginning of 2005, all series are seasonally adjusted using this method. Section 4 covers the procedure of seasonal adjustment at the CBS, that is, the estimation of: prior adjustment factors for trading day and festival effects (4.1), seasonal factors (4.2), seasonally adjusted series (4.2.1), and trend (Section 4.3). Finally, we discuss concurrent seasonal adjustment (Section 4.4.1) and seasonal adjustment of composite (aggregate) series (Section 4.4.2).

The second part of the publication presents tables of the festival and trading day prior adjustment factors, and the seasonal factors for the year 2007. These are provided for more than 400 time series that are currently published in the Monthly Bulletin of Statistics and in other publications of the CBS.

The third part shows diagrams illustrating the original, festival and trading day adjusted, seasonally adjusted, and trend series, for the main socio-economic indicators in Israel’s economy for 2003-2007. In addition, diagrams for festival and trading day, seasonal, and irregular factors, as well as the percentage change on the previous month (quarter) in the trend are included for the same series and the same period.

The appendix presents data on Jewish festival dates and working day variation in Israel for the years 1980-2010.
2. Methodology

2.1 Time Series and Its Components

A time series is a sequence of observations ordered in time, \( X_1, X_2, ..., X_t, ..., X_T \); the interval between times \( t \) and \( t+1 \) being fixed and constant throughout.

The time series collected by the CBS are statistical records of a particular social or economic activity, like industrial production, person-nights in tourist hotels, road accidents. They are measured at regular intervals of time, usually monthly or quarterly, over relatively long periods. This allows the disclosure of patterns of behavior over time, to analyze them and place the current estimates into a more meaningful and historical perspective.

Notionally, a time series can be decomposed into a number of fundamental components, each of which has its own distinguishing character. In a simple model, the original data at any time point (denoted by \( O_t \)) may be expressed as a function \( f \) of three main components: the seasonality \( S_t \), the trend-cycle \( C_t \), and the irregularity \( I_t \), that is

\[
O_t = f(S_t, C_t, I_t)
\]

Diagram 1 shows a graphic representation of the original series (non-seasonally adjusted series) of Departures of Israelis Abroad by Air, monthly, from January 2000 to March 2004. It can be seen from the diagram that the series exhibits regular seasonal effect, with departures high in summer and low in winter.

Depending mainly on the nature of the seasonal movements of a given series, several different models can be used to describe the way in which the components \( C_t, S_t \), and \( I_t \) are combined to compose the original series \( O_t \). There are three basic seasonal models in which X-12-ARIMA can decompose the series and identify and remove the seasonality. The choice of model depends on whether it is more appropriate to consider the trend, seasonal and irregular components to be proportional to each other or largely independent of each other.

Specifically, the multiplicative model treats all three components as dependent on each other; that is, the seasonal oscillation size increases and decreases with the level of the series. In contrast, the additive model assumes that the absolute sizes of the components of the series are independent of each other and, in particular the size of the seasonal oscillations is independent of the level of the series. The pseudo-additive model treats the seasonal and irregular components as independent of each other but dependent upon the level of the trend.

In the multiplicative and the pseudo-additive models, the trend-cycle \( C_t \) is measured in the same units as the original series \( O_t \), and the seasonal \( S_t \) and irregular components \( I_t \) are expressed as percentages, so they are centered about 100. In the additive model all three components are measured in the same units as the original series and the seasonal and irregular components vary about 0. In Diagram 2, three examples of monthly time series are presented. For each series, only one seasonal decomposition model is appropriate.
Diagram 2.a: **Multiplicative model.** This is the most frequently used model for time series. As can be seen in the diagram, the magnitude of the seasonal fluctuations is proportional to the level of the series. This model assumes that the trend, seasonal and irregular components are represented by a multiplicative function so that the observed series at time \( t \) may be modeled by:

\[
O_t = C_t \times S_t \times I_t
\]  

Although most series at the CBS are adjusted multiplicatively there are some exceptions. Series that include extremely small values or zeros cannot be adjusted using a multiplicative model, so an additive or a pseudo-additive model is used.

Diagram 2.b: **Additive model.** This is the most appropriate model for series where the magnitude of the seasonal component does not appear to be affected by the level of the series. The series is decomposed in this case as follows:

\[
O_t = C_t + S_t + I_t
\]

Diagram 2.c: **Pseudo-additive model.** This model provides the best seasonal adjustment for series with extreme seasonal variations that is some months (quarters) having extremely small values or zeros and the remaining months (quarters) appearing to have a multiplicative seasonality. The seasonal and irregular components are expressed in percentages. The model combines the elements of both the additive and multiplicative models as follows:

\[
O_t = C_t + C_s (S_t - 1) + C_i (I_t - 1) = C_t \times (S_t + I_t - 1)
\]

Each one of the three main components of the time series, that is the **seasonality** \((S_t)\), the **trend-cycle** \((C_t)\), and the **irregularity** \((I_t)\), displays distinctly different characteristics and will be discussed below.
2.1.1 Seasonality ($S_t$)

The seasonal component represents intra-year, monthly or quarterly, fluctuations that recur every year with more or less the same timing and intensity. Thus, they create an annual pattern of changes, a seasonal pattern. Some examples of this kind of influences are the considerable increase of residents departing abroad in June, July and August, the significant drop in prices of clothing in February-March and September.

Causes for seasonal fluctuations include natural factors (climatic variations: summer, winter, rainfalls), administrative or legal measures (starting and ending dates of school year, fiscal year) and social/cultural/traditional, and calendar-related effects that are stable in annual timing (e.g., public fixed holidays such as Christmas, Valentine’s Day). Calendar-related
systematic effects associated with the dates of moving holidays like Passover and the Jewish New Year (Rosh Hashanah) are not seasonal in this sense, because they occur in different calendar months depending on the dates of the holidays.

Diagram 3 presents examples of seasonal patterns:

Diagram 3.a illustrates the seasonal pattern of one series: Departures Abroad of Israelis by Air, for 2003. The values of the estimated seasonal component are called seasonal factors. The seasonal factors, estimated using the multiplicative model, are measured on a percentage scale. Factors greater than 100 percent indicate a high season (the original series is greater than the trend), and factors less than 100 percent indicate a low season. As the diagram shows, the series exhibits a distinctive pattern – typically high in June, July and August, the months most people take vacations, and lower in November, December, January and February.

Diagram 3.b shows the different seasonal patterns of 12 series: monthly series of incoming tourism to Israel by country of citizenship, estimated using the multiplicative model. It is seen that the twelve seasonal patterns vary substantially both in shape and magnitude.

2.1.1.1 Range of Seasonality
Seasonal variation can be very large or relatively small. The difference between the highest (peak) and the lowest (trough) of the annual seasonal factors, expressed in percentages, is called the range of seasonality, and is a measure of the magnitude of the seasonal variation. Using the range of seasonality, the Israeli series were classified into three groups: a) high seasonality (range greater than 50 %), b) intermediate (20-50 %), and c) low (less than 20%). Diagram 4 illustrates examples for each group. Note the changes in the values in the y-axis, of the seasonal factors, for each example.

2.1.1.2 Stable and Moving Seasonality
The seasonal pattern may be constant or evolve in time during the analyzed period. If it is constant or remains almost the same in time, in magnitude and shape, then it is said that the series presents stable seasonality. In the case that it evolves, that is the seasonal pattern changes gradually over time, in amplitude, shape, or both amplitude and shape, it is said that the series has moving seasonality. There are many causes for the latter: seasonal pattern may gradually evolve as economic behavior, economic structures, technological advances, and institutional and social arrangements change. For example, a gradual decrease is observed in the magnitude of the seasonal component for agriculture series caused by technological changes that reduce the effect of weather on growth and sales of fruits and vegetables. In a composite (aggregate) series, that is the series is composed of sub-series, it may occur due to change in weights of sub-series.
Table 1 presents the seasonal factors of the monthly series Generation of Electricity (in millions of kilowatts per hour), for January 1996 to December 2003. The series is decomposed using the multiplicative model, so the seasonal factors are expressed in percentages. A seasonal factor of 100.0 denotes that there is no seasonal effect. The yearly average of the factors is 100.0, meaning that all the seasonal influences occur during a single year. The table shows that the series has moving seasonality, with gradual change for almost all months in the period 1996-2003. The seasonal factors for January decrease monotonically from 109.1 to 106.7 and for August they increase from 115.3 to 122.0.
The seasonality becomes more significant in March, June, July, September, October and November, whereas in December the seasonality weakens over time. For the rest of the months, February, April and May, the seasonal effects remain nearly the same.

### 2.1.2 Trend-Cycle ($C_t$)

Many time series exhibit a tendency to grow or to decrease fairly steadily over quite long periods of time. This pattern is identified as **trend**. Another interpretation of the trend is that it represents the underlying direction of the series lasting for many years, and obtained after excluding the seasonal and the irregular influences. The trend captures the long-term behavior of the series and therefore its movements are smooth and gradual. A quasi-periodic oscillation may be sometimes observed around the trend, characterized by alternating periods that are generally longer than one year, **cycles**. An example of it is the business cycle, during which the socio-economic activity alternately expands and contracts.

The identification of trend has always posed a serious statistical problem. No convenient analytical method has been found to separate the long-term trend from the cycles. Therefore, they are treated as a single component, **trend-cycle** ($C_t$). The trend-cycle stems from factors such as population growth, technological development, and economic or security situations.

A **trend break**, or level shift, is defined as an abrupt but sustained change in the level of a series. The annual seasonal pattern is not changed, but is simply at a different level. The reasons for a trend break can be changes in concepts and definitions of a survey, collection method, technology, economic behavior, legislation, social traditions, etc. For example, in Israel, due to the security situation, a strong trend break was detected as of October 2000 for all series of incoming tourism and person-nights of tourists in tourist hotels. **Diagram 5** presents an example of the trend-break detected in the series Tourist Arrivals by Air.

Trend breaks are a problem for seasonal adjustment as they can cause distortions in the estimates of all components of the series. Hence, if a trend break is detected in the data it should be handled before the seasonal adjustment. The X-11-ARIMA solution is to create a back series that maintains the seasonal pattern in the original data but shifts the level so it is consistent with the new level of the series.
This new series is used to seasonally adjust the series for the period after the trend break. For the data before the trend break the seasonal adjustment is carried out as usual. In the X-12-ARIMA method, a level shift variable can be included in the model to account for a sharp and sustained change in the level of the series. In any case, the final seasonally adjusted series will show a discontinuity at the beginning of the trend break. (An explanation on the seasonal adjustment methods is provided in Section 3).

2.1.3 Irregularity ($I_t$)

This component represents the variation remaining after adjusting the original series for trend and seasonality. It comprises three parts:

a) **Calendar changes**, such as those associated with the moving dates of religious festivals (which might simply result in the transfer of socio-economic activity between successive months like March-April and September-October), changes in the daily (weekday) composition of months of the same year, and between the same month of different years (e.g. the number of Sundays, Mondays). These influences can be estimated and forecast based on past experience, since the calendar structure is known.

b) **Extreme one-time events**, such as wars, strikes, special weather conditions and economic instability. Usually, these events cannot be forecast and it is difficult to estimate their effect.

c) **Residual irregularity**, arising from measurement errors, response errors and other errors.

2.1.4 Trading Day and Festival Effects ($P_t$)

As indicated in Section 2.1.3, the irregular factors ($I_t$) may be decomposed into sub-components: the changes in the number of trading days and festival dates (also called calendar effects), and the remaining irregularity. Therefore model (1) may be extended as follows:

$$I_t = P_t \times E_t,$$

and thus

$$O_t = C_t \times S_t \times P_t \times E_t \quad \text{(2)}$$
where $P_t$ is the adjustment factor for the calendar effects (changes in the festival dates and the number of trading days in a month) and $E_t$ is the residual variation caused by all other influences. (For further explanation on the estimation of $P_t$, see Section 4.1).

Diagram 6 summarizes the decomposition of a time series into its components. The series analyzed is Departures of Israelis Abroad, by Air, and the period presented is January 2000 to March 2004. A multiplicative seasonal model is used for the decomposition. The diagram provides an illustration of the meaning of seasonal adjustment showing separately the components of the series, that is, the prior adjustment for festivals and trading day, the seasonal, the irregular and the trend-cycle.

The unadjusted data (original series) presents the actual economic events that have occurred, while the seasonally adjusted data and the trend-cycle estimate represent an analytical elaboration of the data designed to show the underlying movements that may be hidden by the festival and trading day, seasonal and the irregular variations. These three sets of data provide useful information about the economy and they should be presented to the users.

The original series ($O_t$), of Diagram 1, is shown in the top panel of diagram 6. The prior adjustment factors for festival and trading day ($P_t$), in the second panel, highlight the adjustments for the Spring and Autumn Jewish holidays, with a relatively high effect in 2002. In that year both festivals dates fell close to their earliest possible starting date, that is, the series is estimated to be high by 25% in March and 3% in September, and low in April by 30% and by 10% in October. The seasonal factors ($S_t$) in the next panel show, that the structure observed in Diagram 3.a for 2003 repeats itself in other years. Analyzing the seasonality of the series it can be seen that it has high and stable seasonality. That is, the original data is higher by 70% in the peak months of July and August and low by 35% in the winter months of November, December, January and February. The irregular factors ($E_t$), shown in the fourth panel, indicate low unexplained effects, not larger than 7%. Observe the particularly strong one-time extreme irregular effect of 10% in November 2001, after the terror attack in New York and 15% in March 2003, the month that started the war in Iraq. In the last two panels the seasonally adjusted series, ($S_A_t$) and the trend ($C_t$) are displayed. The fifth panel presents the seasonally adjusted series. Notice that it is smoother than the original series as the regular influences of festival and trading day, and seasonality have been removed, although the irregular component still remains, especially visible in November 2001 and March 2003. In order to eliminate also the irregular component, the trend-cycle is estimated, using the seasonally adjusted data, thus obtaining a smoother an easy interpretable series that is presented in the sixth panel.
Diagram 6. Example of the decomposition of a time series into its components, Departures Abroad of Israel by Air.
3. X-11-ARIMA and X-12-ARIMA Methods

The U.S. Census Bureau devised a general approach to seasonal adjustment in 1965, called the X-11 method, which became the standard method used by many government statistics offices around the world. A major development of the method was made by Statistics Canada in 1980, with the seasonal adjustment method named X-11-ARIMA. Its most important improvement is that it allows the user to augment the observed series, before seasonal adjustment, with forecasts (and backcasts) values from ARIMA models. The use of forecast extensions generally results in smaller revisions of the seasonal adjustments, especially at the end of the series, on average; better trend estimation at the end of the series. In addition, it includes the estimation of indirect seasonal adjustment of series that are aggregate of multiple component series, and better diagnostics for assessing the quality of the seasonal adjustment. In 1996, the U.S. Census Bureau developed an enhanced version of the X-11-ARIMA called X-12-ARIMA. X-12-ARIMA’s major enhancements include new X-11-ARIMA adjustment options, new and better diagnostics, new modeling capabilities especially for handling calendar effects, and improved user interface.

The basic algorithm of X-11 is common to both the X-11-ARIMA and X-12-ARIMA methods. It uses moving averages to estimate the main components of the series: trend and seasonality. Moving averages are essentially weighted averages of the observations. They are nonparametric, simple and especially flexible in their application. It is possible to construct a moving average that has good properties in terms of trend preservation, elimination of seasonality and noise reduction. Section 3.1.2 explains in detail the types of moving averages used by the mentioned methods.

Finally, it is worth mentioning that the X-11-ARIMA and the new X-12-ARIMA methods are the most widely used programs for seasonal adjustment in government statistics offices, some other organizations and universities. In Europe, some government agencies and statistics organizations use the package TRAMO/SEATS, a model-based approach that is described in Gomez and Maravall (2000).

3.1 Moving Averages

There are various types of arithmetic averages. In a simple average all the values contribute equally to the average, that is, they have an equal weight in the calculation. In relation to time series data, an average may be applied to the whole series to produce a single averaged value, or an average may be applied to successive overlapping sub-periods of the series, thereby producing a series of averaged values. In the latter case, the results are produced from a simple moving average. For example, applying a 13 simple moving average to a time series is carried out as follows: first calculate the simple average of the first 13 consecutive observations, then move along the series one observation, dropping out of the calculation span the first observation and bringing in the fourteenth observation, and calculate the simple average of these 13 observations. The process of moving along the series one observation at a time and taking the 13-term average is repeated, until there are no further time series observations to bring into the 13-term span.

A moving average can be of any length and can take on any weighting pattern as long as the weights are positive and sum to 1. As different weighting patterns tend to give rise to moving averages with different characteristics, moving averages are often classified on the basis of their associated weighting patterns. We mentioned above the simple moving average, based on uniform weights. If the weights are not uniform the moving average is said to be non-simple. A moving average may be also described as being either symmetric or asymmetric, depending on the form of its weighting pattern. In particular, a moving average is said to be symmetric if the weighting pattern used to calculate it is symmetric about the center of the averaging span, otherwise it is said to be asymmetric. In a centered simple moving
average, the results of applying symmetric moving averages are placed in the center of each average span, in order not to introduce phase shifting in the new series.

3.1.1 Definition

We define a moving average \( M(O_r) \) of order \( p + f + l \) and weights \( W_k \) as:

\[
M(O_r) = \sum_{k=-p}^{f} W_k \cdot O_{t+k}.
\]

Thus, the value at time \( t \) of the original series is therefore replaced by a weighted average of \( p \) “past” values of the series, the “current” value, and \( f \) the “future” values of the series.

- The quantity \( p + f + 1 \) is called the moving average order.
- When \( p = f \), that is when the number of observations in the past is the same as the number of observations in the future, the moving average is said to be centered.
- If, in addition, \( W_{-k} = W_k \) for any \( k \), the moving average \( M \) is said to be symmetric; otherwise is said to be asymmetric.

Generally, with a moving average of order \( p + f + l \) calculated for time \( t \) with \( p \) observations in the past and \( f \) observations in the future, it will be impossible to smooth out the first \( p \) observations and the last \( f \) observations of the series.

A composite simple moving average of order \( P \times Q \) is obtained by composing two simple moving averages in succession. First, a moving average of order \( Q \), with weights equal to \( 1/Q \) is applied, and then a simple moving average of order \( P \), with weights equal to \( 1/P \) is used. The order of the resulting composite moving average will be \( P + Q - 1 \), and is denoted \( M_{P,Q} \). The resulting weights, for time \( t \) in the center of the span are in the form of:

\[
(I, 2, ..., P - 1, P, P, ..., P, P - 1, ..., 2, 1) / P \times Q,
\]

where the number of times \( P \) appears in the above parenthesis is \( 1+|P-Q| \).

| Table 2. Examples of the weighting patterns of symmetric 13-term moving averages |
|-------------------------------|-------------------------------|
| Weight                       | Month                        |
|                              | \( t-6 \) | \( t-5 \) | \( t-4 \) | \( t-3 \) | \( t-2 \) | \( t-1 \) | \( t \) | \( t+1 \) | \( t+2 \) | \( t+3 \) | \( t+4 \) | \( t+5 \) | \( t+6 \) |
| Simple                       | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              | 1/13              |
| Non-simple (composite simple)| 1/24              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/12              | 1/24              |
| Non-simple (Henderson)       | -0.019            | -0.028            | 0                 | 0.006             | 0.147             | 0.214             | 0.240             | 0.214             | 0.147             | 0.066             | 0                 | -0.028            | -0.019            |

Table 2 presents three examples of the weighting patterns of 13-term symmetric simple and non-simple moving averages for the central value \( t \). In the second row all weights are equal, in the third row, the first and last terms have half the weight of the others, and in the last row
only the observations that are equidistant from the central value \( t \) have equal weights.

3.1.2 Symmetric Moving Averages Used in X-11

In the X-11 method, common to the X-11-ARIMA and X-12-ARIMA methods, symmetric moving averages play an important role. In order to avoid loss of information at the beginning and end of the series, they are supplemented by ad hoc asymmetric moving averages. Specifically, X-11-ARIMA and X-12-ARIMA use ARIMA modeling techniques to extend the original series, so that a smaller number of asymmetric moving averages are applied.

Two examples of the main symmetric moving averages used by the X-11 method are:

**Example 1.** The preliminary trend moving averages \( M_{2\times12} \) and \( M_{2\times4} \)

For monthly series, the estimation of the preliminary trend is obtained using a \( 2 \times 12 \) composite simple moving average on the original series \( (O_t) \). This average is also known as a centered 12-term moving average. The resulting weights are:

\[
M_{2\times12} (O_t) : (1,2,2,2,2,2,2,2,2,2,2,2,1)/24.
\]

For quarterly series, the estimation of the preliminary trend is obtained using a \( 2 \times 4 \) composite simple moving average on the original series \( (O_t) \). This average is also known as a centered 4-term moving average. The resulting weights are:

\[
M_{2\times4} (O_t) : (1,2,2,2,1)/8,
\]

as shown in the last row of Table 3. It is seen these weights can be expressed as sums of 1’s (equal weights) divided by the total number of appearances of each value, and thus are easily computed. Also note that the number of rows in the table corresponds to \( P=2 \) spans.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Quarter</th>
<th>Sum of Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t-2 )</td>
<td>( t-1 )</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>For second span</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Final Weights</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**Example 2.** The seasonal moving averages \( M_{3\times3} \) and \( M_{3\times5} \)

The estimation of the seasonal component is obtained, in most cases, using a \( 3 \times 3 \) and then a \( 3 \times 5 \) composite moving average on the seasonal-irregular \( (S_t \times I_t) \) component for each month (or quarter) separately. The weights for these moving averages are:
\[ M_{3,5}(S_i \times I_t) : (1,2,3,2,1)/9 \]

and

\[ M_{3,5}(S_i \times I_t) : (1,2,3,3,2,1)/15, \]

respectively.

### 3.1.3 Properties of Moving Averages

- **Smoothing** - All moving averages smooth out the series to which they are applied. However, different moving averages will smooth out the variations differently. The smoothing process is also referred to as **filtering**.

- **Timing** - Symmetric centered moving averages have the characteristic of placing the turning point at the correct timing. It is not possible to apply these averages for the first and last observations of the series. In this case asymmetric non-centered moving averages provide the solution. Asymmetric moving averages may cause time phase shifting; for example, delay in the timing of major turning points.

- **Tracking** - A simple moving average can accurately reproduce only straight-line segments. Most of the major socio-economic time series may not be well approximated by straight lines, because the series may have peaks or troughs, and periods of accelerating or declining growth. These are better approximated by the use moving averages that preserve mixtures of linear, quadratic and cubic functions, e.g. the Henderson moving averages, as explained below.

### 3.1.4 Symmetric Henderson Moving Averages

Robert Henderson published a formula for the computation of the weights of his moving averages. The weights are designed, so that the long-term trend estimates are as smooth as possible and reproduce a wide range of curvatures that may include peaks and troughs. The Henderson moving averages can be computed for any number of odd terms. In the CBS, we mainly use a 13-term Henderson moving average for the monthly series and a 5-term Henderson moving average for quarterly series. The weights for these moving averages are presented in Table 4.

The main properties of these averages are:

a) **Eliminate almost all the short irregular variations (shorter than 6 months).** The weighing pattern was designed to give smoothest results (minimizes the sum of squares of the third differences of the smoothed series for any number of terms). In the smoothed series obtained by these moving averages, irregularity will be efficiently dampened down. Since sampling errors, residual seasonality and residual movable holidays and trading day effects can be characterized by cycles of up to 6 months, the filtering process will substantially eliminate them.

b) **Conserve the amplitude of the waves of long periods (12 months or longer).** Many economic time series exhibit cyclical patterns that are longer than one year. For example, business cycles during which the socio-economic activity alternately expands and contracts. These patterns are not necessarily regular, but they do follow rather smooth patterns of upswings and downswings. However, frequently there is not enough regularity to allow their reliable prediction. It is convenient to explain historical behavior in terms of such cyclic movements that remain in the smoothed series. The seasonal pattern for a monthly series is represented by the annual cycle of 12 months. About 85 percent of the strength of the fundamental seasonal cycle of 12 months would remain, if the series being smoothed had not first been seasonally adjusted, thus removing the seasonal cycle. It is for this reason that the
Henderson moving averages are never applied to data that contain fundamental seasonality.

c) **Do not distort the timing and the sharpness of the turning points.** As explained above, symmetric moving averages do not cause time phase shifting, thus place correctly the timing of the turning points. The Henderson moving averages have been designed to reproduce not only straight lines segments but also quadratic and cubic functions, and as a result better approximating of the sharpness of turning points as compared to those achieved by straight lines segments.

d) **Provide relatively stable and generally robust estimates.** The revisions to the changes of the trend series (excluding the last six observations) are smaller than the revisions to the seasonally adjusted series, even for relatively volatile series.

Table 4 presents the weighting patterns of symmetric Henderson moving averages, for monthly series and quarterly series.

**Table 4.** Symmetric Henderson weights for the trend of a monthly series and a quarterly series

<table>
<thead>
<tr>
<th>Time</th>
<th>( t-6 )</th>
<th>( t-5 )</th>
<th>( t-4 )</th>
<th>( t-3 )</th>
<th>( t-2 )</th>
<th>( t-1 )</th>
<th>( t )</th>
<th>( t+1 )</th>
<th>( t+2 )</th>
<th>( t+3 )</th>
<th>( t+4 )</th>
<th>( t+5 )</th>
<th>( t+6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly series</td>
<td>-0.019</td>
<td>-0.023</td>
<td>0</td>
<td>0.066</td>
<td>0.147</td>
<td>0.214</td>
<td><strong>0.240</strong></td>
<td>0.214</td>
<td>0.147</td>
<td>0.066</td>
<td>0</td>
<td>-0.023</td>
<td>-0.019</td>
</tr>
<tr>
<td>Quarterly series</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Diagram 7, the weights of the 13-term Henderson moving average for a monthly series from Table 4 are plotted.
3.2 Basic Algorithm of X-11

The X-11 method is based on an iterative estimation of the time series components, using appropriate arithmetic moving averages. The method handles the decomposition and seasonal adjustment of monthly and quarterly series. In estimating the seasonally adjusted series the input is the original series prior adjusted for festival and trading day effects. (For explanation of the estimation of festival and trading day effects see Section 4.1).

In order to identify and remove the variations associated with the seasonal effects, the program uses a series of moving averages and smoothing calculations to decompose the original series \( O_t \) into trend \( C_t \), seasonal \( S_t \), and irregular \( I_t \) components. While the series can be decomposed into these three components, a good estimate of the seasonality cannot be made until the trend has been removed, and likewise a reliable estimate of the trend cannot be computed until the seasonality has been removed. To overcome this problem a recursive approach is used. Preliminary estimates of the trend are used to obtain preliminary estimates of the seasonal variation, which in turn are used to get better estimates of the trend and so on. In order to reduce or eliminate the influence of extreme observations, the program detects and modifies the extreme values used in the estimation of the seasonal factors.

Let a monthly time series \( O_t \) be decomposed using the multiplicative model (1). The main stages of the multiplicative version of the moving average filtering procedure of the X-11 method, assuming prior adjusted original series, are extraction of the initial estimates; computation of the final estimates of seasonal factors, forecast seasonal factors and seasonally adjusted series; and finally the extraction of the final estimates of the trend and of the irregular factors. Three stages and their corresponding sub-stages are described next and are presented in Table 5.

**Stage 1: Initial estimates**

**Substages: 1.1 to 1.5:**

1.1: Crude trend-cycle

A first estimate of the trend-cycle is obtained by applying a \( M_{2\times12} \) moving average to the original series \( O_t \).

\[
C_t^{(1)} = M_{2\times12}(O_t)
\]

This relatively crude moving average is used in order to eliminate constant monthly seasonality. The moving average should reproduce, at best, the trend-cycle component while eliminating the seasonal component and minimizing the irregular component.

1.2: Detrended series from crude trend (unmodified seasonal-irregular component)

The ratio between the original series and the estimated trend gives the first estimate of the detrended series, that is, the unmodified seasonal-irregular factors \( S_t \times I_t \)

\[
S_t^{(1)} = O_t / C_t^{(1)}
\]

1.3: Crude “biased” seasonal factors

The seasonal component is estimated by smoothing the seasonal-irregular component **one month at a time**; first we smooth the values corresponding to the month January, then the values corresponding to February and so on. The calculation is carried out using a 5-term moving average, \( M_{3\times3} \), for each month, to estimate the crude (preliminary) seasonal factors \( S_t^{(0)} \) from the seasonal-irregular factors \( S_t^{(1)} \). These seasonal factors are called biased because their yearly averages may not be equal to one hundred percent (recall that in a multiplicative model the seasonal factors are measured in percentages and their mean should
be equal to 100). Hence, we have:

\[ S_t^{(0)} = M_{3\times3}(S_t^{(1)}) \]

1.4: Crude “unbiased” (normalized) seasonal factors via centering

The crude (preliminary) seasonal factors \( S_t^{(0)} \) are then normalized, so that the average of each 12-month period equals approximately to one hundred. That is, each seasonal factor is divided by a centered 12-term moving average of the preliminary seasonal factors, resulting in \( S_t^{(1)} \).

\[ S_t^{(1)} = S_t^{(0)} / M_{2\times12}(S_t^{(0)}) \]

1.5: Preliminary seasonally adjusted series

The first estimate of the seasonally adjusted series \( S_t^{(1)} \) is obtained by removing from the original series \( O_t \) the estimate of the seasonal component \( S_t^{(1)} \)

\[ S_t^{(1)} = O_t / S_t^{(1)} \]

Stage 2: Final estimates of seasonal factors and seasonally adjusted series

Substages: 2.1 to 2.5:

Here the strategy used is similar to that followed in Stage 1 (in 1.1 and 1.2). The differences are that for the trend estimation of substage 2.1, a 13-Henderson moving average \( H_{13} \) is used on the preliminary seasonally adjusted series of 1.5; and for the estimation of the final seasonal factors, of substage 2.3, a 7-term moving average \( M_{3\times5} \) is used.

2.1: Henderson trend

The estimate of an intermediate trend-cycle \( C_t^{(2)} \) is obtained by applying a 13-term Henderson moving average \( H_{13} \) to the preliminary seasonally adjusted series \( S_t^{(1)} \) from 1.5.

\[ C_t^{(2)} = H_{13}(S_t^{(1)}) \]

2.2: Detrended series from Henderson trend (unmodified seasonal-irregular factors)

The trend component \( C_t^{(2)} \) from 2.1, is removed from the analyzed series \( O_t \) so as to provide a final estimate of the seasonal-irregular component \( S_t^{(2)} \)

\[ S_t^{(2)} = O_t / C_t^{(2)} \]

2.3: Final “biased” seasonal factors

The next iteration of the seasonal component is similar to 1.3 but applying a 7-term moving average, \( M_{3\times5} \), to each month of the seasonal-irregular component \( S_t^{(2)} \) to obtain

\[ S_t^{(2)} = M_{3\times5}(S_t^{(2)}) \]

2.4: Final “unbiased” seasonal factors

Calculate the final “unbiased” seasonal factors via centering as in 1.4
\[ S_t^{(3)} = S_t^{(2)} / M_{2\times12}(S_t^{(2)}) \]

A more detailed description of the estimation of the seasonal factors, including the estimation of forecast seasonal factors, is presented in Section 4.2.

**2.5: Final seasonally adjusted series**

Estimate the final seasonally adjusted series \( SA_t^{(2)} \) by dividing the original series \( O_t \) by the final seasonal factors \( S_t^{(3)} \)

\[ SA_t^{(2)} = O_t / S_t^{(3)} \]

**Stage 3: Final estimates of trend, and irregular factors**

**Substages 3.1 to 3.2:**

**3.1 Final trend from Henderson trend filter**

The estimate of the final trend-cycle \( C_t^{(3)} \) is obtained by applying a 13-term Henderson moving average to the final seasonally adjusted series \( SA_t^{(2)} \) from 2.5

\[ C_t^{(3)} = H_{13}(SA_t^{(2)}) \]

**3.2 Final irregular factors**

The final irregular factors \( I_t \) are the ratio between the final seasonally adjusted series from 2.5 and the final trend estimate from 3.1

\[ I_t = SA_t^{(2)} / C_t^{(3)} = O_t / (S_t^{(3)} \times C_t^{(3)}) \]

Note that the final trend estimates from 3.1 are not the final trend estimates used by the CBS. For explanation of the improved method for the estimation of the trend see Section 4.3.1.

**Table 5** presents the basic algorithm of the X11 method.

**3.3 ARIMA Models**

The ARIMA part incorporated into the X-11 ARIMA program plays an important role in the estimation of concurrent seasonal factors and seasonal factor forecasts. In order to reduce the effect of asymmetric filtering, the X-11-ARIMA method **extrapolates the prior adjusted original series** \( O_t^{(1)} \) **or the original series** \( O_t \), if no prior adjustment is carried out, with Box Jenkins AutoRegressive Integrated Moving Average (ARIMA) models. When a series is extended with extra data, the filters applied by X-11-ARIMA to estimate the concurrent (and forecast) seasonal factors are closer to the filters used for central observations. Consequently, the degree of reliability of the extended series for current estimates is greater than that of the unextended series, and the magnitude of the revisions is significantly reduced.
ARIMA models bring together two basic concepts in extrapolation: autoregression and moving averages. In the word ARIMA, AR stands for “Autoregressive”, MA for “Moving Average” and the I, for “Integration”. The integration part of ARIMA is indispensable since stationary models, which are fitted to the differenced data, have to be summed or “integrated” to provide models for non-stationary data. In the Box and Jenkins notation, the general multiplicative ARIMA model for series with seasonality (called also the SARIMA model) of order \((p,d,q)(P,D,Q)s\) is expressed as:

\[
\phi_p(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D O_i^{(1)} = \theta_q(B)\Theta_Q(B^s)e_i
\]

where \(s\) is the length of the seasonal period, \(B\) is the backshift operator \(B(Y_i) = Y_{i-1}\), \(\phi_p\) and \(\theta_q\) are polynomials in \(B\) of degrees \(p\) and \(q\) respectively, \(\Phi_P\) and \(\Theta_Q\) are polynomials in \(B^s\) of degrees \(P\) and \(Q\) respectively, \(d\) and \(D\) are the orders of ordinary and seasonal differences respectively and \(e_i\) is a white noise process. Note that all the polynomials should satisfy stationarity and invertibility conditions. For further details see Box and Jenkins (1976).

In addition, X-12-ARIMA is enhanced with a RegARIMA module that enables modeling of the original series with explanatory variables as well as allowing ARIMA structures for the
errors. Such models are used to preadjust the series before seasonal adjustment by removing effects such as trading day, moving holidays, and outliers, and to forecast the prior adjusted series (Findley, et al., 1998).

4. Seasonal Adjustment Procedure at the Central Bureau of Statistics

Section 3.2 describes the basic algorithm used by X-11 to obtain estimates of seasonal factors, seasonally adjusted series, trend and irregular factors. Recall that we have suggested the extended model (2), in Section 2.1.4, that decomposes the irregular component \(I_r\) into the festival and trading day component \(P_r\) and the remaining irregularity \(E_r\). To accommodate this model we perform three runs of X-11-ARIMA (two runs of X-12-ARIMA) as follows:

**First run of X-11-ARIMA: Pre-Adjustment** – the prior adjustment factors for the Jewish festivals and trading day effects in Israel \(P_r\) are estimated using the irregular factors \(I_r\) obtained from the first run of the seasonal adjustment method on the original series \(O_r\). See Section 4.1.

**Second run of X-11-ARIMA: Seasonal Adjustment** – the prior adjustment factors are applied to the original series and then the characteristics of the prior adjusted series are studied so that the seasonal adjustment method can be tailored to the behavior of the particular series. In having made the appropriate choices, the program is run again on the series prior adjusted \(O_r / P_r\) and then the seasonal factors \(S_r\) and the seasonally adjusted series \(S_A_r\) are calculated.

**Third run of X-11-ARIMA: Trend Estimation** – in order to eliminate the influence of the irregularity in the seasonally adjusted series and illustrate the development of the series, the improved trend-cycle \(C_r\) is estimated using a third run of the program.

When using X-11-ARIMA, the estimation of the component \(P_r\) is carried out using an external program. In contrast, when using X-12-ARIMA the component \(P_r\) can be estimated using an internal procedure of the program that accepts user-defined variables. Thus, the first two stages of X-11-ARIMA are carried out in one run of the X-12-ARIMA method. The second run of X-12-ARIMA corresponds to the third X-11-ARIMA run. Figure 1 provides a schematic illustration of the sequential X-11-ARIMA runs, indicating on the left hand side the corresponding X-12-ARIMA runs.

4.1 Prior Adjustment Factors for Trading Day and Festival Effects

A method was developed at the CBS for the simultaneous estimation of the moving festival dates and the number of trading days effects. The analysis of a time series starts with estimation of the effects of festivals and trading days. These precalculated estimates are then used for prior adjustment of the series. The prior adjusted original series is subsequently analyzed using the seasonal adjustment programs mentioned above. As seen in Figure 1, the combined linear effect of the festivals and the trading day in a month (or quarter) are estimated from the irregular factors \(I_r\), in the same month, obtained in the first run of X-11-ARIMA. This calculation is carried out using an external SAS program. With X-12-ARIMA a similar approach is applied but the estimation is carried out using its built-in procedure. The regression models used for estimating these effects are described below.
4.1.1 Explanatory Variables

The explanatory variables used for estimating the effect of trading days and Jewish festival dates are:

a) The effect of the trading day is measured by the daily activity variables \(X_i\), \(i = 1, \ldots, 7\), where \(i\) indicates the day of the week starting from Sunday. Thus, \(X_1\) is the number of Sundays in the month, \(X_2\) is the number of Mondays in the month, and so on. The variable \(X_6\) indicates the number of Fridays and festival eves, and \(X_7\) the number of Saturdays and festival days. Note that the number of Sundays, Mondays, ..., Thursdays, does not include festival eves or festivals that fall on these days.

b) The effect of the Jewish festival dates is measured by the following variables:

\(X_8\) The Jewish festival date is defined as the number of days between the actual starting
date and the earliest possible starting date of the Passover festival and the Jewish New year (in March-April and September-October). See explanation in the Appendix.

\( X_9 \) **The number of intermediate festival days – Hol Hamoed** (in March-April and September-October).

c) **The effect of the Easter holiday (Christian)** is measured by:

\( X_{10} \) The Easter date for the effect of Easter holiday (in March-April) is defined in a similar way to that of the Jewish festival date variable. This variable is used only for Tourist Arrivals and Person-nights of Tourist Hotels in Israel series.

The trading day and festival effects are estimated from a multiple regression model in which the dependent variable is the transformed irregular factor \((I_i)\) and the independent variables are functions of \(X_i\) to \(X_{10}\) as defined above (for example, we use the differences \(X_1 - X_7\),..., \(X_6 - X_7\), rather than \(X_i\), as the sum of \(X_1,...,X_7\) is equal to the number of days in a month).

### 4.1.2 Regression Models for Monthly and Quarterly Series

**a) Monthly series:**

For monthly series with at least 8 years of data, a regression model is fitted separately for 5 groups of months:

- Group 1 – the winter months (November to February)
- Group 2 – Passover months (March and April)
- Group 3 – the months preceding the summer vacation (May and June)
- Group 4 – the summer vacation months (July and August)
- Group 5 – the New Year, Yom Kippur and Succoth festival months (September and October).

For series with 6 to 8 years of data, these effects are estimated separately for only three groups of months: the winter months (November to February), the summer months (May to August) and the festival months (March, April, September and October).

For short monthly series of 5 to 6 years, the combined effect of the trading day and the moving festival date is estimated using all months as one group.

**b) Quarterly series:**

For quarterly series with more than 10 years of data, a regression model is fitted separately for 2 groups of quarters:

- Group 1 – quarter I (January to March) and quarter II (April to June)
- Group 2 – quarter III (July to September) and quarter IV (October to December)

For short quarterly series of 7 to 9 years, the effect of the trading day and the Jewish festivals is estimated all quarters as one group.

For monthly series with less than 5 years of data, and for quarterly series with less than 7 years of data, the trading day and festival effects are not estimated. Therefore, the seasonal adjustment for these series will be carried out on the original series without prior adjustment.

When fitting a regression model it is tested whether groups of months (quarters) may be pooled. In the case where they can not be pooled, then the regression model is applied separately to each group of months. If pooling is acceptable then the effects are estimated using all months (quarters) in the pooled group.
It should be pointed out that in all cases the estimation of the festival effect is carried out for each festival month (quarter) separately. That is, the model enables to split the festival effect independently between the two festival months (quarters).

The prior adjustment factors ($P_i$) for the simultaneous effect of festivals dates and the numbers of trading days are estimated from the final model. As the daily composition of the months (quarters) and the festival dates are known in advance, forecast prior adjustment factors can be obtained for the following year.

Table 6 presents examples of prior adjustment factors for 2003 for two series: Manufacturing Production Index and Sales Value Index of Chain Stores. In both examples, all months are influenced by trading days, and March-April and September-October are influenced by the combined effect of moving festival dates and trading days.

As it can be seen, for both series in the example, the factors for May, August and November are below average. This can be explained because months having more Fridays and Saturdays are expected to have lower levels of activity. In 2003, May had 5 Thursdays, 5 Fridays and 5 Saturdays, and August had 5 Fridays, 5 Saturdays and 5 Sundays. On the other hand, the factors for July and December are above average. Those months had less Fridays and Saturdays and therefore more trading (working) days, so we expect them to present a higher level of activity. The daily composition of those months was: July had 5 Tuesdays, 5 Wednesdays and 5 Thursdays, and December had 5 Mondays, 5 Tuesdays and 5 Wednesdays. Notice that he combined trading day and festival effects strongly influence the festival months. In particular, see the factors in the Manufacturing Production Index in September (107.6) and in Sales Value Index of Chain Stores in March (91.6), April (105.5) and September (102.9).

Diagram 8 shows the prior adjustment factors for the Manufacturing Production Index for the period January 2000 to March 2004 (the factors for 2003 are those presented in Table 6). It is seen that the pattern changes from year to year because the daily composition of each month changes from year to year, and because the festival dates move in a pattern that repeats itself after 19 years.

<table>
<thead>
<tr>
<th>Series</th>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing Production Index</td>
<td></td>
<td>100.9</td>
<td>99.1</td>
<td>99.4</td>
<td>100.9</td>
<td>98.2</td>
<td>100.1</td>
<td>102.4</td>
<td>97.0</td>
<td>107.6</td>
<td>99.8</td>
<td>98.6</td>
<td>101.5</td>
</tr>
<tr>
<td>Sales Value Index of Marketing Networks</td>
<td></td>
<td>101.5</td>
<td>100.0</td>
<td>91.6</td>
<td>105.5</td>
<td>98.0</td>
<td>100.0</td>
<td>101.6</td>
<td>97.1</td>
<td>102.9</td>
<td>101.0</td>
<td>96.6</td>
<td>101.2</td>
</tr>
</tbody>
</table>
4.1.3 Prior Adjusted Original Series

Once the trading day and festival factors are estimated, the original series is prior adjusted for these effects by dividing the original series by the estimated factors:

\[ O^{(1)}_t = O_t / P_t \]

4.2 Seasonal Factors (S_t)

The estimation of the seasonal factors (S_t) is based on the seasonal-irregular factors (SI_t). The latter are obtained by removing the preliminary trend-cycle component (C_t) from the analyzed prior adjusted original series, that is:

\[ SI_t = O^{(1)}_t / C_t \]

The seasonal factors are then estimated by smoothing the seasonal-irregular factors (SI_t) one month at a time; first we smooth the values corresponding to the month January, then the values corresponding to February and so on. The smoothing process is done, by X-11, using seasonal moving averages that are weighted arithmetic averages applied to the same month (quarters) over several years. These moving averages are applied to the seasonal-irregular factors, to separate the seasonal from the irregular. Extreme values of the seasonal-irregular factors are detected and replaced before the final smoothing process. Thus, the estimation is based on the corrected values. The moving average used can be selected depending on the characteristics of the series. In X-12-ARIMA one can select from the following moving averages: \( M_{3x3}, M_{3x5}, M_{3x7}, M_{3x9}, M_{3x15} \) or stable seasonal average. The latter is calculated by a simple average of all SI_t values for each calendar month separately.

In practice, and for most of the series seasonally adjusted at the CBS, two seasonal moving averages are used iteratively: in Stage 1 (substage 1.3) a 5-term moving average, \( M_{3x5} \), and in Stage 2 (substage 2.3) a 7-term moving average, \( M_{3x7} \).

Here we describe in detail the substages 1.3 and 2.3 of the estimation of the seasonal factors presented in section 3.2.
Substage 1.3

- Estimate the preliminary seasonal component by smoothing the seasonal-irregular factors, each month separately, using a $M_{3x3}$ moving average.

- Normalize the seasonal factors in such a way that, for one year of observations, their average is roughly equal to 100.0 (for a multiplicative model).

- Estimate the irregular component by removing the initial normalized seasonal factors from the seasonal-irregular component.

- Calculate a five-year moving standard deviation of the estimates of the irregular component and test the irregulars in the central year of the five-year period. Remove the values beyond 2.5 and recalculate a five-year moving standard deviation.

- Assign a weight of 1 (full weight) to the irregulars within 1.5 standard deviations, a linearly graduated weight, between 0 to 1, to the irregulars between 1.5 to 2.5 standard deviations, and 0 to the irregulars beyond 2.5 standard deviations.

- The values of the seasonal-irregular factors whose irregular do not receive a full weight are considered extreme and are adjusted and replaced. The replacement is the weighted average of the value itself with its assigned weight, and the two values before and the two values after it, having full weights.

- Estimate again the seasonal factors by smoothing the modified (corrected for extreme values) seasonal-irregular factors, each month separately, using the $M_{3x3}$ moving average.

Substage 2.3

- Substage 1.3 is repeated, but this time a $M_{3x5}$ moving average is used to obtain the final seasonal factors.

The forecast seasonal factors, $S_{n_{j+1},j}$ for one year ahead, where $n_j$ is the last available year for a given month $j$, are estimated:

a) using the extrapolated original series from the ARIMA model and the seasonal moving averages, or

b) if the ARIMA option is not applied or no model is selected, by a simple linear projection on the basis of the last two final seasonal factors for a given month.

$$S_{n_{j+1},j} = S_{n_{j},j} + \frac{(S_{n_{j},j} - S_{n_{j-1},j})}{2}$$

Table 7 presents the final unmodified seasonal-irregular factors for 1996-2003 (Table 7.a), the final modified seasonal-irregular factors for 1996-2003 (Table 7.b), the final seasonal factors for 1996-2003 (Table 7.c), and the forecast seasonal factors for year 2004 (Table 7.d), for the series Residents Departing Abroad by Air. In Table 7.a the values of the final unmodified seasonal-irregular factors that were detected as having extreme irregular component values and that were modified in Table 7.b, are presented in bold. For example, in 2003, six of the values were detected as outliers and modified with most noteworthy modification for June, where the value of 115.5 was replaced by 102.7. Table 7.c presents the final factors that were used to estimate the final seasonally adjusted series. In Table 7.d, the one-year ahead forecast seasonal factors are shown. They were estimated by applying seasonal moving averages to the extrapolated original series, by an ARIMA model.
Table 7. Examples of (a) final unmodified seasonal irregular factors, (b) final modified seasonal irregular factors, (c) final seasonal factors, (d) forecast seasonal factors, in percentages, Departures Abroad of Indians by Sex.

(a)

<table>
<thead>
<tr>
<th>Year</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>67.1</td>
<td>67.7</td>
<td>94.5</td>
<td>94.3</td>
<td>98.6</td>
<td>96.5</td>
<td>157.3</td>
<td>179.7</td>
<td>128.9</td>
<td>103.0</td>
<td>66.5</td>
<td>81.8</td>
<td>101.0</td>
</tr>
<tr>
<td>1997</td>
<td>70.3</td>
<td>58.2</td>
<td>90.0</td>
<td>92.9</td>
<td>78.2</td>
<td>100.1</td>
<td>162.4</td>
<td>169.5</td>
<td>136.5</td>
<td>107.2</td>
<td>64.1</td>
<td>74.7</td>
<td>99.2</td>
</tr>
<tr>
<td>1998</td>
<td>68.4</td>
<td>60.7</td>
<td>87.7</td>
<td>90.7</td>
<td>86.8</td>
<td>97.3</td>
<td>165.3</td>
<td>172.9</td>
<td>129.9</td>
<td>109.8</td>
<td>64.0</td>
<td>69.7</td>
<td>99.8</td>
</tr>
<tr>
<td>1999</td>
<td>69.0</td>
<td>62.6</td>
<td>86.3</td>
<td>94.3</td>
<td>87.0</td>
<td>110.8</td>
<td>142.7</td>
<td>171.2</td>
<td>126.3</td>
<td>103.7</td>
<td>68.2</td>
<td>70.1</td>
<td>100.9</td>
</tr>
<tr>
<td>2000</td>
<td>68.7</td>
<td>61.2</td>
<td>90.2</td>
<td>93.9</td>
<td>87.4</td>
<td>102.9</td>
<td>189.1</td>
<td>167.2</td>
<td>129.7</td>
<td>107.8</td>
<td>63.8</td>
<td>67.9</td>
<td>99.3</td>
</tr>
<tr>
<td>2001</td>
<td>65.0</td>
<td>62.4</td>
<td>87.9</td>
<td>92.7</td>
<td>91.5</td>
<td>100.7</td>
<td>171.0</td>
<td>176.4</td>
<td>128.1</td>
<td>100.6</td>
<td>59.4</td>
<td>66.6</td>
<td>100.3</td>
</tr>
<tr>
<td>2002</td>
<td>65.3</td>
<td>64.0</td>
<td>85.6</td>
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- 27 -
Diagram 9 presents the final modified seasonal-irregular factors (blue line) from Table 7.b and the final seasonal factors (red line) from Table 7.c, for each month separately for the period 1996-2003. Note the different scales of the y-axis.

4.2.1 Final Seasonally Adjusted Series \((SA_t)\)

Seasonal adjustment is the process of identifying and removing the seasonal component from the time series. This allows consecutive months (quarters) to be comparable, and thus provides a reliable estimate of the short-term change in the series. A seasonally adjusted series is also called the first approximation to the trend-cycle. In cases where the series is only
influenced by seasonality, the seasonally adjusted series \((SA_t)\) is estimated by dividing the original series \((O_t)\) by the final seasonal component \((S_t)\):

\[
SA_t = O_t / S_t
\]

In practice, most original series are affected by festival and trading days influences. For these cases the final seasonally adjusted series \((SA_t)\) is obtained by the removal of the estimated effects of both the trading day and the movable holiday influences \((P_t)\) and the seasonal influences \((S_t)\) from the original series \((O_t)\):

\[
SA_t = O_t / (P_t \times S_t)
\]

Table 8 illustrates the process of seasonal adjustment for the Departures Abroad of Israelis by Air series. In this example the original series \((O_t)\) is influenced by trading day and festival effects, and by seasonality.

Diagram 10 presents the original and the seasonally adjusted series shown in the first and last rows of Table 8. The diagram highlights the strong influence of seasonality and festival and trading day variations as seen in the original series. The seasonally adjusted series is much smoother because those influences were removed. The smaller contribution of irregularity can be observed in some months. Notice that in February and March 2003 the influence of the Iraq war caused a decrease in the number of Israelis leaving the country during those months.

<table>
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<th>Table 8. Example of estimation of seasonally adjusted series, Departures Abroad of Israelis by Air, 2003</th>
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<td>Prior adjustment factors ((P_t)) (in percentages)</td>
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<td>Prior adjusted series ((O_t^{(P)} = O_t / P_t)) (in thousands)</td>
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<tr>
<td>Seasonal factors ((S_t)) (in percentages)</td>
</tr>
<tr>
<td>Seasonally adjusted series ((SA_t = O_t^{(P)} / S_t)) (in thousands)</td>
</tr>
</tbody>
</table>
4.3 Trend Estimates

By definition, the seasonally adjusted series \((SA_t)\) consists of the underlying trend \((C_t)\) and the irregular component \((E_t)\):

\[
SA_t = O_t / (P_r \times S_r) = C_t \times E_t
\]

Therefore, it is a preliminary estimate of the trend. Whether the seasonally adjusted data are a good approximation of the trend or not depends on the relative contribution of the irregular component to the monthly movements, as compared to the contribution of the trend. The Months for Cyclical Dominance (MCD) is a measure based on a comparison of these relative contributions. The MCD is defined as the minimum number of months, \(m\), required on average for the relative change in the trend-cycle to "dominate" the irregularity of the series. That is, the smallest number of months for which the percent change of the irregularity, averaged over the time points in the interval \([m+1,T]\), is smaller than the respective average for the trend-cycle. Formally, the MCD is the first \(m\) for which:

\[
\frac{\sum_{t=m+1}^{T}|E_t - E_{t-m}|/E_{t-m}}{\sum_{t=m+1}^{T}|C_t - C_{t-m}|/C_{t-m}} < 1.0
\]

The Quarters for Cyclical Dominance (QCD) is defined in a similar way for quarterly series, indexed by \(q\). The measures MCD and QCD may be used to determine the length of the moving average needed to smooth the seasonally adjusted series, and X-11 calculates the smoothed seasonally adjusted series using these measures. Notice that the seasonally adjusted series may contain extreme values that will influence the smoothed series obtained.

An attempt to discern the trend-cycle requires that the influence of the irregularity in the seasonally adjusted series be dampened in such a way that the estimates provide timely and reliable indicators of trend behavior. X-11 estimates a final trend by default as follows: First, a modified original series, with extreme values detected and replaced, is estimated. Then a final modified seasonally adjusted series is calculated by dividing the modified original series by the final prior adjustment and seasonal factors. Last, the trend is estimated using the
Henderson moving averages on the final modified seasonally adjusted series. This trend is not published by the CBS; instead, the trend is estimated by an improved method.

4.3.1 Improved Method for the Estimation of the Trend

The trend-cycle in use at the CBS is estimated using X-11-ARIMA or X-12-ARIMA together with an improved method proposed by Statistics Canada (Dagum, E. B., 1996). This improved method for the estimation of the trend is based on the symmetric Henderson moving averages, but applied to the final seasonally adjusted series modified twice for extreme values and extended with forecast values. As mentioned before, the main reason for modifying the seasonally adjusted series prior to the estimation of the trend is that it may contain extreme values that may lead to false interpretations of changes in the trend-cycle, or to detection of false turning points. Another important aspect of this improved method is that it also deals with the end-point problem, that is, it estimates the last 6 monthly observations or the last 2 quarterly observations of the trend by applying symmetric moving averages to the modified seasonally adjusted series extended with forecast values.

This improved method consists of:

a) First modification of the final seasonally adjusted series for very extreme values (the corresponding values to the irregular factors with a standard deviation value greater than 2.5). If a value of the seasonally adjusted series is detected as such, then it is replaced by the value of the final trend-cycle estimate (obtained in 3.1 of Stage 3 in the basic algorithm of X-11).

b) Extension of the modified seasonally adjusted series with six forecast values, for monthly series, or two forecast values for quarterly series, using a simple ARIMA model.

c) Second modification of the final seasonally adjusted series, this time using stricter criteria than the one used in (a). The standard deviation limits for the identification and replacement of extreme values are 0.7 and 1.0 standard deviation. Those values for which the irregular component is greater than 1.0 standard deviation are assigned 0 weights and those between 0.7 and 1.0 are assigned a linearly graduated weight.

d) Applying the Henderson filter on the twice modified seasonally adjusted series obtained from (c) and extended with forecast from (b).

The advantages of the improved method are as follows:

a) Reduces the number of unwanted ripples in the trend-cycle that may lead to the detection of false turning points.

b) Detects the turning points at the correct timing, with a delay of, at most, three to four months.

c) Reduces the size of the revisions of the trend-cycle at the end of the series.

The limitations of the trend estimates:

In general, the trending procedure gives rise to revisions of the estimates of the trend-cycle for the most recent part of the series. Therefore, the estimates of the trend-cycle for the most recent part of the series should be interpreted with care, that is, they are provisional and subject to change. This is especially important for seasonally adjusted series that are highly irregular.

In particular, emphatic statements about the presence of a current trend turning point based on the behavior of the last period should be postponed until more data is analyzed. There are two main reasons for caution. First, usually it is not possible to distinguish between an outlier and a change in direction based on a single observation at the end of the series. It is only after a lag of several observations (three to four months) that the real change in the trend comes to
light. Second, the forecasts at the end of the series assume that the most recent underlying trend in the series will persist. Consequently, the last estimated trend values continue to point in the direction of the former trend.

To assist the analysis of the recent and current trend behavior we, at the CBS, present the trend-cycle estimates together with the seasonally adjusted series from which they were derived. This helps to analyze the magnitude and direction of the differences between the two series, the differences being an estimate of the impact of the irregular component. In the diagrams representing the two series, it should be noticed that an asterisk appears for the last three points of the trend in a monthly series (one asterisk for the last quarter of quarterly series), indicating that the trend estimates for these time points are provisional and are subject to substantial revisions as new data are incorporated into the estimation of the trend-cycle.

Diagram 11 shows an example of a seasonally adjusted series and its corresponding trend-cycle estimate.

Presenting the seasonally adjusted series and the trend-cycle in the same chart highlights the overall development of the two series over time. As we can see the trend-cycle series is smoother because the irregularity that was present in the seasonally adjusted series was successfully removed.

Diagram 12 presents examples of trend-cycle estimates for Indices of Manufacturing for the period January 2000 to August 2004.
4.4 Special Issues

4.4.1 Concurrent Seasonal Adjustment

Concurrent Seasonal Adjustment means that the seasonally adjusted series is calculated anew each month (quarter) on the basis of data that includes the current (new) observation. In general, new data contribute new information about changes in the seasonal pattern that preferably should be incorporated as early as possible. When concurrent seasonal adjustment is applied to a series, the seasonally adjusted values for the whole series, including the most recent month (quarter), are obtained directly from the seasonal adjustment procedure without the use of forecast seasonal and prior adjustment factors. Empirical studies have shown that the revisions to the seasonally adjusted data are smaller for series seasonally adjusted using the concurrent method.

The forecast seasonal factors \((S_t)\) and the forecast prior adjustment factors \((P_t)\) for 2007, presented in the Tables part of this publication, are not used for the seasonal adjustment of series that are adjusted concurrently. They are given here only for the sake of completeness and in order to demonstrate the extent of the influence of seasonality and festivals and trading days on the series. We note that concurrent seasonal adjustment is applied to all series published in the Monthly Bulletin of Statistics and in other publications of the CBS.

Table 9 presents the results of concurrent seasonal adjustment for Manufacturing Production Index. A first seasonal adjustment was performed with data from January 1995 to November 2003, when the datum for November 2003 was obtained. Then, as a new observation, December 2003, became available; a second seasonal adjustment was carried out for data including this observation. It is seen that the changes for previous months did not exceed the 0.8 points.

4.4.2 Seasonal Adjustment of Composite (Aggregate) Series

An aggregate series (composite) is a series that is composed of several sub-series (components). The direct seasonal adjustment method seasonally adjusts the composite series and the component series independently.

The indirect seasonal adjustment method seasonally adjusts each component series and then sums the seasonally adjusted component series to obtain the seasonally adjusted composite series.

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<td>123.9 123.7 122.1 122.7 122.7 122.2 122.7 123.0 122.3 127.6 125.0 125.7</td>
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</table>
In the past, all CBS composite series were seasonally adjusted using the direct method. At the present, the indirect method for calculating the seasonally adjusted composite of series from a certain system of series has been adopted, as a result of a thorough study in composite series and their components. The study established that for some composite series more reliable results are obtained by using the indirect method. The advantage of this procedure is that the changes in the composite series can be broken down into the changes of the component series. Those component series for which the relative contribution of seasonality is low and/or the contribution of irregularity is high, are included in the sum as unadjusted original series.

Table 10 presents an example of direct and indirect seasonal adjustment for the composite (aggregate) quarterly series Unemployed Persons, by Sex. The component series are Unemployed Males and Unemployed Females. It is seen, that the total indirect seasonally adjusted series is the sum of the two seasonally adjusted components. The differences between the direct and indirect seasonally adjusted series are relatively small, see Diagram 13. In general, it is easier to explain the changes in the composite series using the indirect approach.

|                | Year |            |            |            |            |            |            |            |
|----------------|------|------------|------------|------------|------------|------------|------------|
|                | 2001 | I          | II         | III        | IV         | 2002       | I          | II         | III        | IV         | 2003       | I          | II         | III        | IV         | 2004       | I          | II         |
| Males          |      | 104.7      | 114.4      | 125.2      | 133.6      | 142.0      | 140.3      | 135.4      | 130.2      | 142.5      | 137.8      | 142.2      | 144.7      | 137.6      | 138.8      |            |            |
| Females        |      | 107.8      | 107.2      | 114.7      | 121.6      | 122.3      | 119.7      | 128.0      | 124.3      | 131.1      | 135.4      | 138.9      | 144.6      | 149.2      | 148.9      |            |            |
| Total indirect |      | 212.5      | 221.6      | 241.9      | 255.2      | 264.3      | 260.0      | 263.4      | 263.5      | 276.6      | 273.2      | 284.1      | 289.3      | 286.8      | 287.7      |            |            |
| Total direct   |      | 210.2      | 222.2      | 241.2      | 255.5      | 263.6      | 261.3      | 263.1      | 261.8      | 276.4      | 274.9      | 282.3      | 289.0      | 286.4      | 288.3      |            |            |
At the CBS, composite (aggregate) series for which the **indirect seasonal adjustment** method has been adopted are presented below:

- **Migration and Tourism:** Departures Abroad of Israelis, Returns of Israelis from Abroad.
- **Balance of Payments:** Credit, Debit.
- **Foreign Trade:** Imports, Exports, Trade Deficit.
- **Labor Force Survey:** Employed Persons (by sex), Unemployed Persons (by sex).
- **Construction:** Construction of Dwellings, by Initiator of Construction.
- **Commerce:** Revenue Index of Trade and Services.
- **Hotels:** Person-nights in Tourist Hotels.
- **Transport:** International Air Traffic of Passengers via Ben-Gurion Airport, Road Accidents with Casualties, Casualties in Road Accidents.

The remaining aggregate series, for which the composite series and their component series are seasonally adjusted separately, that is, for which the **direct seasonal adjustment** method is used, are as follows:

- **Migration and Tourism:** Returns of Israelis within Three Months, Tourist Arrivals by Air.
- Prices: Consumer Price Index, Consumer Price Index Excluding Housing, Consumer Price Index Excluding Housing and Vegetables and Fruit.
- Labor and Wages: Average Weekly Work Hours per Employed Person by Industry based on Labor Force Survey; Employee Jobs, Wages based on Reports to the National Insurance Institute.
- Manufacturing: Manufacturing Production Index, Employees Index, Index of Man-hours Worked, Sales (revenue) at Current Prices, Paid Hourly Wage per Employee.
- Commerce: Sales Value Index of Chain Stores, Private Consumers Credit Cards Purchases.
- Hotels: Person-nights of Tourists, Person-nights of Israelis in Tourist Hotels.

5. The Publication

5.1 Description of the Tables

The tables of this publication present the forecast factors for 2007 of the seasonal, prior adjustment and combined factors for all series seasonally adjusted at the CBS. The tables should be read as follows:

- The forecast seasonal factors are given in the first row and are denoted by $S$.
- The forecast prior adjustment factors for the festival and trading day effects, if calculated, are given in the second row and are denoted by $P$.
- The forecast combined factors for seasonal adjustment, i.e. the product of the above factors, are given in the third row and are denoted by $S \times P$.
- The range of seasonality is calculated as the difference between the highest and the lowest annual seasonal factors.
- The measures MCD (Months for Cyclical Dominance) or QCD (Quarter for Cyclical Dominance), defined in Subsection 4.3, are given for each series.
- The heading of each system of series includes the alphabetical symbol of the chapter and the serial number of the relevant table as they appear in the Monthly Bulletin of Statistics.

Table 11 shows the forecast seasonal, prior adjustment and combined factors for 2004 of the monthly series Departures Abroad of Israelis by Air. Notice the special symbols that indicate the type of adjustment.

5.2 Description of the Diagrams

This publication also includes diagrams for the main socio-economic indicators in Israel's economy. For each indicator, two sets of diagrams that show the patterns of the series for the years 2003-2007 are given.

- The first set of diagrams illustrates the original series, festival and trading day adjusted series, seasonally adjusted series, and the trend estimated by the improved method. The festival and trading day adjusted series is the original series after removing the influence of these effects. The difference between the original series and the seasonally adjusted series is the combined estimated effects of the seasonal, trading day
and the movable holiday influences that have been removed from the original series. The difference between the seasonally adjusted series and the trend is the irregular effect. It should be noted that an asterisk appears for the last three points of the trend in a monthly series, in order to indicate that the trends are provisional and are subject to revision. Similarly, for quarterly series an asterisk appears for the last quarter.

- **The second set of diagrams** illustrates festival and trading day factors, the seasonal factors, the irregular factors and, in addition, the percentage change on the previous month in the trend.

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<tbody>
<tr>
<td></td>
<td></td>
<td>Jan</td>
</tr>
<tr>
<td>Seasonal</td>
<td>$S$</td>
<td>64.7</td>
</tr>
<tr>
<td>Prior adjustment</td>
<td>$P$</td>
<td>100.0</td>
</tr>
<tr>
<td>Combined</td>
<td>$S \times P$</td>
<td>64.7</td>
</tr>
</tbody>
</table>

### 5.3 Comparison to the Previous Publication

In comparing the forecast seasonal factors and forecast prior adjustment factors, (Internet only), for 2007 with those published for 2006, the following should be considered:

a) The forecast of the seasonal factors for 2007 is based on a longer period (data for 2006 and part of 2007).

b) The forecast of the prior adjustment factors for the effects of festival and trading day for 2007 are calculated based upon the same period as the seasonal factors.

c) As of October 2000 (due to the security situation in Israel) and then again as of July 2007 (due to the second Lebanon war) the original data recorded in the series of incoming tourism (total and by country) and Person-nights in Tourist Hotels (total and by locality) were extremely irregular. Mainly, abrupt changes in the levels of original data, i.e. a trend breaks in the series were detected. Therefore, calculation of seasonally adjusted data and trend data were carried out after adjusting the data up to September 2000 and June 2006, respectively, to the low levels received during the first months of each crisis.

d) For the first time, in this publication, seasonal and prior adjustment factors are presented for the series: Employed Persons by District, Unemployed Persons by District and Private Consumers Credit Card Purchases.

e) The diagrams for the main socio-economic indicators are presented for the years 2003-2007.
Type of Adjustment (Special Symbols)

\[ S = \text{Seasonal factors} \]
\[ P = \text{Festival and trading day prior adjustment factors} \]
\[ S \times P = \text{Combined factor for seasonal adjustment (product of the seasonal and the festival and trading day prior adjustment factors).} \]
5.4 Bibliography


